The origin and evolution of spiral arms in disk galaxies is a fundamental problem in astrophysics. One theory to explain the spiral arms is swing amplification. In a differentially rotating disk, a leading density pattern rotates to a trailing one due to the shear. During the rotation, the leading mode is amplified into spiral arms due to the self-gravity if the Toomre’s Q is 1–2.

In our previous work, we investigated the pitch angle of spiral arms by the local linear theory and simulations [1]. The derived pitch angle formula well agrees with N-body simulations.

We extended the our previous work and calculated the pitch angle, the wavelengths, and amplification factor of the most amplified mode [2]. The derived formulae of the pitch angle, the radial wavelength, the azimuthal wavelength, and the amplification factor are

\[
\tan \theta_{JT} = \frac{1}{2\pi} \left(1 + \frac{2.095}{Q^{5.3}}\right)^{-1} \frac{\kappa}{A},
\]

(1)

\[
\tilde{\lambda}_{r, JT} = \frac{0.581Q^2 - 1.558Q + 1.547 \Omega^2}{1 + 2.095Q^{-5.3}} \frac{\kappa}{A},
\]

(2)

\[
\tilde{\lambda}_{\phi, JT} = (3.653Q^2 - 9.789Q + 9.721) \left(\frac{\Omega}{\kappa}\right)^2,
\]

(3)

\[
D_{\text{max}} = 0.0657 \exp\left(7.61 \frac{\kappa A}{Q} \frac{\Omega^2}{\kappa}\right),
\]

(4)

where \(\kappa\) is the epicycle frequency, \(\Omega\) is the orbital frequency, \(A\) is the Oort constant, \(Q\) is the Toomre’s Q.

Figure 1 shows the results. We confirmed that the formulae derived from the linear theory well agree with N-body simulations.

Next, in order to understand the physics of the swing amplification, we revisited the swing amplification model [3]. We carefully considered its derivation and we found that the naive treatment of the model leads to breakdown of the model in the strong shear case such as a Keplerian rotation. Therefore we modified the model for avoiding the breakdown. Using the modified model we investigated the motion of stars in spiral arms. We found that the phases of the epicycle motion of stars are synchronized during the amplification. Based on the phase synchronization we derived the pitch angle formula by the order-of-magnitude discussion. The phase synchronization may be a key process to understand the swing amplification.

References


Figure 1: Comparison between the theory and the simulations. Circles and pluses denote the results of N-body simulations. The dotted and dashed curves show the estimates from the linear theory with \(Q = 1.5\) and \(Q = 1.8\), respectively. Triangles show the lower and upper limit of the amplification factor from the simulations.