Construction of Gauge-invariant Variables of Linear Metric Perturbations on an Arbitrary Background Spacetime

NAKAMURA, Kouji
(NAOJ)

Perturbation techiques are powerful in many areas of physics and the developments of perturbation theories lead to physically fruitful results and interpretations of natural phenomena. General relativity is one of the theories in which the construction of exact solutions is not so easy. It is also true that some exact solutions well describe our universe, or gravitational field of stars, but these exact solutions by themselves do not describe fluctuations around these exact solutions. Therefore, general-relativistic linear perturbation theory is a necessary technique to clarify the properties of uctuations.

Besides linear perturbations, *higher-order* general-relativistic perturbations also have very wide applications: cosmological perturbations; black hole perturbations; and perturbations of a spherical star. Thus, there are many physical situations to which general-relativistic higher-order perturbation theory should be applied.

As is well known, general relativity is based on the general covariance. Due to this general covariance, the "gauge degree of freedom", which is the unphysical degree of freedom of perturbations, arises in general-relativistic perturbations. To obtain physical results, we have to fix this gauge degree of freedom or extract some invariant quantities of perturbations. This situation becomes more complicated in higher-order perturbations. In some linear perturbation theories, there are so-called gauge-invariant perturbation theories, in which one may treat only variables that are independent of the gauge degree of freedom. These gauge-invariant perturbation theories are useful in many cases. Therefore, it is worthwhile to develop higher-order gauge-invariant perturbation theory from a general point of view.

According to these motivations, we have been discussed the general framework of higher-order general-relativistic gauge-invariant perturbation theory and proposed a procedure to find gauge-invariant variables for higher-order perturbations on an arbitrary background spacetime whose metric is given by g_{ab}. This proposal is based on the following single assumption:

Assumption: Suppose that we have the second-rank tensor field h_{ab} whose gauge-transforamtion rule is given by ${}_{\mathcal{Y}}h_{ab} - {}_{\mathcal{X}}h_{ab} = \pounds_{\xi}g_{ab}$, where ξ^a is the generator of the gauge-transformation. Then, there are a second-rank tensor field $\mathcal{H}_{ab}$ and a vector field X^a such that the tensor field hab is decomposed as $h_{ab} = \mathcal{H}_{ab} + \pounds_X g_{ab}$, where the gauge-transformation rules for $\mathcal{H}_{ab}$ and X^a are given by ${}_{\mathcal{Y}}\mathcal{H}_{ab} - {}_{\mathcal{X}}\mathcal{H}_{ab} = 0$ and ${}_{\mathcal{Y}}X^a - {}_{\mathcal{X}}X^a = \xi^a$, respectively.

Confirming that the above assumption in the case of cosmological perturbations is correct, we developed a second-order gauge-invariant cosmological perturbation theory. Through these developments, we find that our general framework of higher-order gauge-invariant perturbation theory is well defined except for the above asumption. Therefore, we proposed this assumption as a conjecture. If this conjecture is true, the higher-order general-relativistic gauge-invariant perturbation theory is completely formulated on an arbitrary background spacetime and has very wide applications.

In Ref. [1], we proposed a scenario of a proof of the above assumption based on the premise that the background spacetime admits Arnowitt-Deser-Misner decomposition. We explicitly constructed the gauge-invariant and gauge-variant parts of linear metric perturbations. Although some special modes are excluded in the proof in Ref. [1], we may say that the above conjecture is almost correct for linear-order perturbations on an arbitrary background spacetime.

Ref. [1] is the full version of our short letter [2]. This short letter is selected by the editors of Classical and Quantum Gravity for inclusion in the "Highlights of 2011-2012" collection on the basis of its novelty and scientfic impact. Furthermore, some ingredients which was not discussed in Ref. [1] was written as an essay for "the 2012 Essay Competition of the Gravity Reasrch Foundation" as Ref. [3] and this essay received an honorable mention.

References

[1] Nakamura, K.: 2013, *Prog. Theor. Exp. Phys.*, **2013**, 043E02.
[2] Nakamura, K.: 2011, *Class. Quantum Grav.*, **28**, 122001.
[3] Nakamura, K.: 2012, *Int. J. Mod. Phys. D*, **21**, 124004.