Explicit-Implicit Scheme for Relativistic Radiation Hydrodynamics

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Relativistic flows appear in many high-energy astrophysical phenomena in which the magnetic field has a crucial role in dynamics. For example, magnetic fields connecting an accretion disk with a central star, or, different points of accretion disks are twisted and amplified due to the differential rotation, launching jets. Also magnetic fields play an important role in accretion disks to transport the angular momentum outward, leading to the mass accretion.

Not merely the observational point of view, but the radiation field is also an important ingredient in the dynamics of relativistic phenomena. The radiation pressure force would play a key role in jet acceleration. Recently, Takeuchi, et al. [1] showed a formation of radiatively accelerated and magnetically collimated jets using non-relativistic radiation magnetohydrodynamic (RMHD) simulations. These magnetic and radiative forces would accelerate jets and outflows to the relativistic speed. But, due to the lack of numerical techniques, RMHD simulations consistently including relativistic effects have not been performed.

The radiation field is described by the radiation transfer equation, which represents time evolutions of the intensity. But it is hard task to solve the transfer equation coupling with the hydrodynamic code due to its complexity and high computational costs. Recently, Farris, et al. [2] proposed numerical schemes to solve general-relativistic radiation magnetohydrodynamic equations. They solved radiation moment equations instead of solving the radiative transfer equations

$$\frac{1}{c}\frac{\partial E_r}{\partial t} + \frac{\partial F_r^j}{\partial x^j} = -G^0, \quad (1)$$

$$\frac{1}{c^2}\frac{\partial F_r^i}{\partial t} + \frac{\partial P_r^{ij}}{\partial x^j} = -G^i, \quad (2)$$

where E_r, F_r^i and P_r^{ij} are the radiation energy density, flux, and stress, respectively.

The radiation and fluids interact each other through absorption and scattering processes. These processes are described by radiation four force G^μ. In their treatment, G^μ is numerically integrated using explicit scheme. Then, a numerical time step Δt should be restricted being shorter than their typical timescales ($\Delta t < \min[t_{ab}, t_{sc}]$) to ensure the numerical stability. Therefore, high computational costs prevent us from studying long term evolutions when the gas is optically thick.

In this paper [3], we propose an numerical schemes to overcome this problem. Governed equations are integrated in time using both explicit and implicit schemes. The former solves an hyperbolic term and latter one treats the gas-radiation interactions through absorption and scattering processes. This method allows us to take a larger time step $\Delta t > t_{ab}$, t_{sc} than that of the explicit scheme.

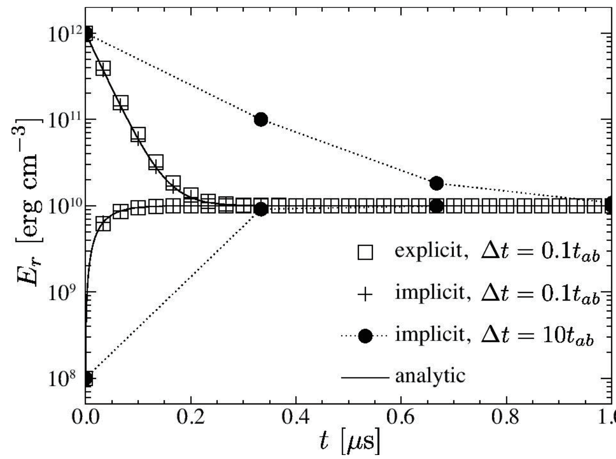

Figure 1: Thermal evolution of radiation energy E_r. Crosses and squares respectively denote results of explicit and implicit schemes, while solid curves do analytical solutions. Filled circles with dotted curves show the results of the implicit scheme with a larger time step of $\Delta t = 10\, t_{ab}$.

In figure 1, we show numerical results that the radiation field approaches to the local thermodynamic equilibrium (LTE) state. We can see that the implicit scheme guarantees that the radiation field stably approaches to LTE with a larger time step Δt. Thus, our scheme drastically reduce computational costs. We believe that our scheme can be applicable to more realistic high-energy astrophysical phenomena.

References

[1] Takeuchi, S., Ohsuga, K., Mineshige, S.: 2010, *PASJ*, **62**, L43.
[2] Farris, B. D., Li, T. K., Liu, Y. T., Shapiro, S. L.: 2008, *Phys. Rev. D*, **78**, 024023.
[3] Takahashi, H. R., Osuga, K., Sekiguchi, Y., Inoue T., Tomida, K.: 2012, *ApJ*, **764**, 122.