

# Particle-Particle Particle-Tree: A Direct-Tree Hybrid Scheme for Collisional $N$ -Body Simulations

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For the time integration of collisional  $N$ -body systems, such as star clusters and systems of planetesimals, the combination of direct summation for force calculation and the individual timestep algorithm has been the standard method for nearly a half century. It is not impossible to combine an individual timestep algorithm and a fast and approximate force calculation. However, it was difficult to achieve good performance on distributed-memory parallel computers for such a scheme.

Figure 1 shows the long-term variation of the relative energy error. In this case, the energy error reaches about  $7.5 \times 10^{-8}$  after the time integration for 10 yr (10 orbital periods), while it is  $9.8 \times 10^{-9}$  for 10 yr (10 periods). In other words, the energy error grows to be 10 times larger as the integration time becomes 1000 times longer. This shows that the growth of the energy error is stochastic, like a random walk. It means that the error is mainly caused by the force error of the tree scheme. The growth of the energy error is, therefore, expected to be slow, and the error is small enough even after long calculations. Figure 2 shows the calculation time per one tree timestep as a function of the total number of particles in system. We used the Intel(R) Core(TM)2 Quad CPU Q6600 (2.4 GHz). It shows that the calculation time increases as  $O(N \log N)$ . Therefore, we reduce the calculation cost from  $O(N^2)$  to  $(N \log N)$ .

We have developed a new hybrid  $N$ -body simulation algorithm for simulating collisional  $N$ -body systems. This new scheme is constructed by combining the tree and direct schemes using a hybrid integrator. The results of test simulations involving the evolution of a planetesimal system show that our new scheme PPPT can drastically reduce the calculation cost, to a level comparable to the cost of a tree scheme with a constant timestep, while keeping the accuracy sufficient for realistic simulations [1].

In principle, our scheme can be used for collisional systems other than planetary systems, such as globular clusters and stars around a supermassive blackhole in the galactic center.

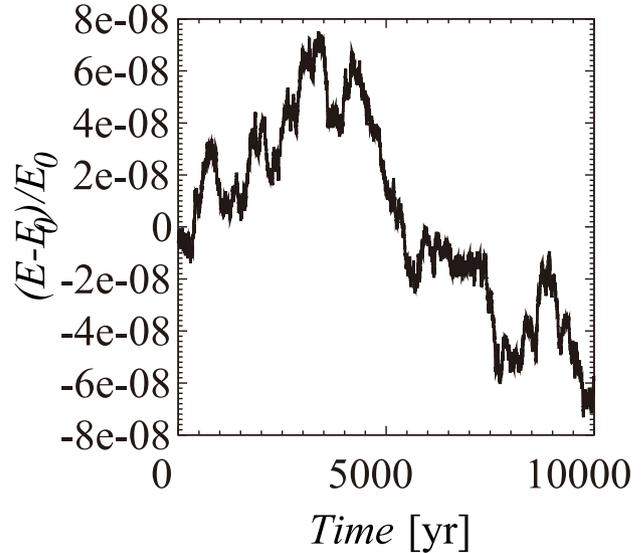


Figure 1: The relative energy error.

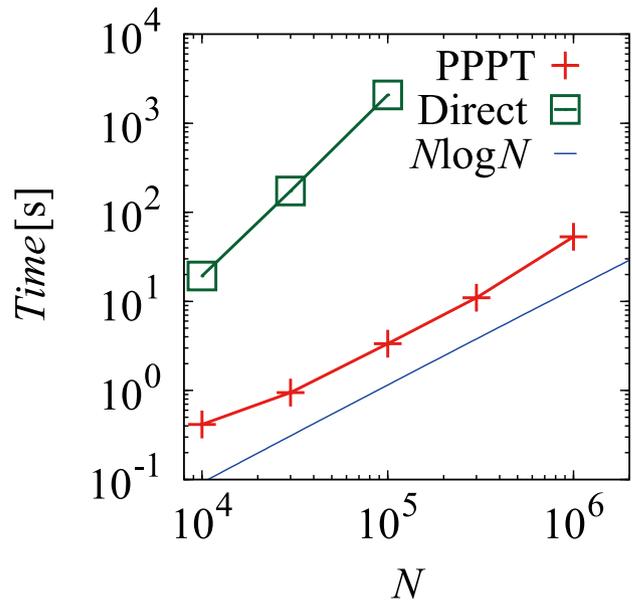


Figure 2: The calculation time plotted against a function of number of particles. The crosses and squares show the results of PPPT and fourth-order Hermite scheme, respectively.

## Reference

[1] Oshino, S., Funato, Y., Makino, J.: 2011, *PASJ*, **63**, 881.