Properties of Weak Measurements through Exact Evaluations

NAKAMURA, Kouji (NAOJ)
NISHIZAWA, Atsushi (Kyoto University)
FUJIMOTO, Masa-Katsu (NAOJ)

Current gravitational wave (GW) detectors have "standard quantum limit" (SQL) due to the uncertainty in quantum mechanics and GW detectors in near future needs to beat this SQL. So, quantum non-demolition measurements are researched in the GW research community and some techniques have been proposed to beat SQL. On the other hand, in quantum information, "quantum measurement theory" is developing and its experiments are carried out. Unfortunately, this development has been almost independent of GW researches. For this reason, we have been exploring applicabilities of techniques in quantum information to GW detectors.

We focused on the "weak measurement" (WM) proposed by Aharonov et al. [1] in 1988. Many experiments of WM are recently carried out and their properties look similar to current GW detectors. Then, we expect that WMs are applicable to GW detectors.

In the usual "measurement" in quantum mechanics, we treat two quantum systems which are called "system" (S) and "detector" (D). We want to measure the observable $\mathbf{A}$ associated with S and D measures $\mathbf{A}$ interacting with S. As a model of D, an one-dimensional quantum system is considered, whose canonical variables are (q, p) ($[q, p] = i$, $\hbar = 1$), and the initial state of D is assumed to be zero-mean Gaussian. The interaction is given by $H = g\delta(t - t_0)p\mathbf{A}$. The state of S before the measurement is $|S_i\rangle = \sum_k \alpha_k |a_k\rangle$, where $\{|a_k\rangle\}$ are eigenstates of $\mathbf{A}$ with eigenvalues $\{a_k\}$. After the interaction at $t = t_0$, the total system is entangled. The measurement outcome of q in D of this state becomes ga_k with the probability $|a_k|^2$. If we know the coupling constant g and if we perform this measurement for many ensemble, we obtain the expectation value $\langle S_i|\mathbf{A}|S_i\rangle$.

On the other hand, in WM, after the interaction between S and D, we concentrate only on the subensemble in which the final state S is $|S_f\rangle$. Then, the expectation value of q in D becomes $g\mathrm{Re}A_w$ within the accuracy of $O(g)$. Here, A_w is called "weak value" defined by $A_w := \langle S_f|\mathbf{A}|S_i\rangle / \langle S_f|S_i\rangle$. From this definition, when $\langle S_f|S_i\rangle \sim 0$, A_w may become large. This result is within $O(g)$ and it implies that the interaction between S and D should be sufficiently weak and g should be small enough. In other words, when we want to measure this small g, it can be measured as an amplified value $g\mathrm{Re}A_w$. This is called the "weak value amplification" (WVA). Aharonov et al. applied this WVA to the sequence of Stern-Gerlach experiments. They claimed that the neutron spin can be measured as 100.

We re-examined the scenario of WMs and concentrated only on the case of measurements of $\mathbf{A}$ with the property $\mathbf{A}^2 = 1$. This property is satisfied in many experimental setups including GW detectors. Then, we have derived the exact expectation value of q of D after the WM as

$$\frac{\langle q\rangle'}{g} = \frac{\mathrm{Re}\langle\mathbf{A}\rangle_w}{1 + \frac{1}{2}\left(1 - |\langle\mathbf{A}\rangle_w|^2\right)(e^{-s} - 1)} \qquad (1)$$

through the all-order evaluation of g [2]. Here, $s := 2g^2\langle p^2\rangle$ is the measurement strength and $\langle p^2\rangle$ is the initial variance of p in D. According to Eq. (1), for a fixed s, there is an optimal overlap $\langle S_f|S_i\rangle$ in which the amplification of the outcome is maximal. The application of Eq. (1) to the sequence of Stern-Gerlach experiments proposed by Aharonov et al. is depicted in Fig. 1.

A recent experiment suggests that Eq. (1) is also experimentally correct [3]. Based on the understanding of WMs which we have obtained, we are now discussing the applicability of WMs to GW detectors.

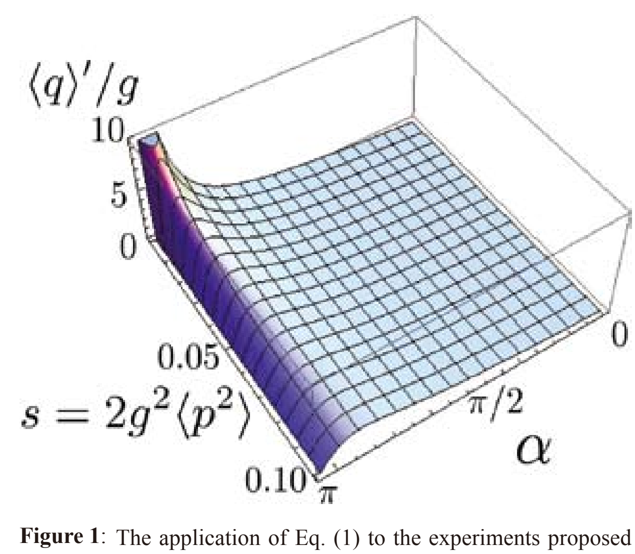

Figure 1: The application of Eq. (1) to the experiments proposed by Aharonov et al. $\alpha \to \pi$ corresponds to $\langle S_f|S_i\rangle \to 0$.

References

[1] Aharonov, Y., Albert, D. Z., Vaidman, L.: 1988, *Phys. Rev. Lett.*, **60**, 1351.
[2] Nakamura, K., Nishizawa, A., Fujimoto, M.-K.: 2012, *Phys. Rev. A*, **85**, 012113.
[3] Iinuma, M., et al.: 2011, *New J. Phys.*, **13**, 033041.