Nonlinear Neutrino Oscillations in Astronomy

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Nonlinear neutrino flavor conversion is investigated by neutrino-neutrino scatterings in the presence of a strong neutrino background and plays an important role in some astronomical sites such as the type II supernovae, the early Universe, and possibly the sites of the gamma ray bursts [1]. In particular, the *forward scattering* and the *flavor exchange* diagrams shown in Fig. 1 play a central role because they undergo coherent superposition and therefore have a dominant contribution [2]. These terms also couple the evolutions of neutrinos with different energies and turn the flavor conversion of neutrinos into a nonlinear many-body problem.

We investigate the flavor evolution of neutrinos undergoing vacuum oscillations and self interactions in an isotropic and homogeneous environment in the absence of a net leptonic background [3]. Assuming a two neutrino mixing scenario, we point out that the resulting flavor evolution has the same dynamics and the same symmetries as i) spins in a system with spin-spin interactions and ii) pair occupation numbers in a system with BCS type pairing [3]. All three of these systems are described by an effective Hamiltonian in the form

$$H = -\sum_{\omega} \omega \mathcal{J}^z_\omega + \mu \vec{\mathcal{J}} \cdot \vec{\mathcal{J}} \, . \tag{1}$$

In the case of neutrinos, the fundamental degree of freedom is the neutrino *isospin* $\vec{\mathcal{J}}$ which is defined in terms of a multiplet of neutrino states [4]. Other relevant quantities and the corresponding physical degrees of freedom are summarized in the following table:

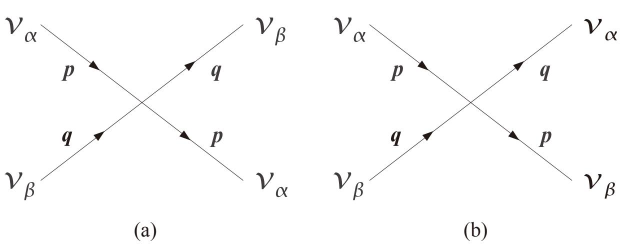

Figure 1: $\nu-\nu$ scattering diagrams corresponding to (a) forward scattering (b) and flavor exchange.

	Neutrino Flavor Evolution	Interacting Spin System	BCS Pairing
$\vec{\mathcal{J}}$	Neutrino isospin	Spin	Pair quasispin
ω	Vacuum oscillation frequency	Gyromagnetic ratio	Single particle energy level
μ	Neutrino density	Spin-spin coupling	Pairing strength

Based on these observations, we point out that the neutrino flavor evolution possesses hidden dynamical symmetries which manifest themselves in terms of the following set of conserved quantities [3,5]:

$$h_\omega = \vec{B} \cdot \vec{J}_\omega + 2\mu \sum_{\omega'(\neq\omega)} \frac{\vec{J}_\omega \cdot \vec{J}_{\omega'}}{\omega - \omega'} \, . \tag{2}$$

The existence of these conserved quantities make the system completely integrable and we can write the exact eigenstates and eigenvalues using the method of Bethe ansatz [3]. Such invariants always simplify the analysis of complex systems. Here we neglected leptonic asymmetry in comparison to the neutrino background which can only be justified in the early Universe. But even when the assumptions which guarantee the existence of constants of motion (including the homogeneity and isotropy of the space) are broken, they can still provide a convenient set of variables which behave in a relatively simple manner depending on how drastic the symmetry breaking is.

The symmetries of the Hamiltonian lead to various collective flavor oscillation modes such as the synchronized oscillations and spectral splits [1,3,6] which may be manifest in a future supernova signal. For example, the analogy between self interacting neutrinos and the BCS pairing system offers an intuitive description of neutrino spectral splits in terms of the conserved quantities. Namely, the method of Bogoliubov transformation borrowed from BCS theory can describe the neutrinos in terms of noninteracting quasi-particle degrees of freedom whose adiabatic evolution from a high density region into the vacuum (like the neutrinos emanated from a supernova) results in the splits of neutrino spectra [3]. A full survey of the symmetries and the associated constants of motion of self interacting neutrinos from the point of view of the emergent collective flavor oscillation modes remains to be an open problem for future research.

References

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