A New Precession Formula

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Modifying J. G. Williams' formulation [1], we developed a new scheme to express the precession and nutation in an arbitrary inertial frame of reference. It gives the precession-nutation matrix as the product of four rotation matrices as

$$NP = R_1(-\epsilon)R_3(-\psi)R_1(\bar{\varphi})R_3(\bar{\gamma})$$

and the precession one similarly as

$$P = R_1(-\bar{\epsilon})R_3(-\bar{\psi})R_1(\bar{\varphi})R_3(\bar{\gamma}).$$

Here (1) $\bar{\varphi}$ and $\bar{\gamma}$ are the angles to specify the location of the ecliptic pole of date in the given inertial frame, (2) ψ and $\bar{\psi}$ are the true and mean ecliptic angles of precession around the ecliptic pole of date and counted clock-wise from the direction to the z-axis of the given inertial frame, respectively, and (3) ϵ and $\bar{\epsilon}$ are the true and mean obliquities of the ecliptic with respect to the $x-y$ plane of the given inertial frame, respectively. Note that $\psi \equiv \bar{\psi} + \Delta\psi$ and $\epsilon \equiv \bar{\epsilon} + \Delta\epsilon$, where $\Delta\psi$ and $\Delta\epsilon$ are the usual nutations in longitude and in obliquity, respectively.

As a result, reference frames referred to the true or mean equator and equinox of date are explicitly described in the ICRF by using the newly introduced precession angles and the usual nutation angles. Although the expression of nutation matrix is unchanged as

$$N = R_1(-\epsilon)R_3(-\Delta\psi)R_1(\bar{\epsilon}),$$

we recommend the usage of the above form of NP instead of preparing P and N separately because of faster evaluation. The formulation is robust in the sense it avoids a singularity caused by finite pole offsets near the epoch. Facing the singularity is inevitable in the pre-2003 IAU formulation.

By using the new formulation, we created a new set of precession formulas [2]. First, we adopted a numerical determination of the motion of ecliptic in DE405 [3] to specify the polynomial expressions of $\bar{\gamma}$ and $\bar{\varphi}$ in the inertial sense. Next, we selected a recent theory of the forced nutation of the non-rigid Earth, SF2001, [4], to express $\Delta\psi$ and $\Delta\epsilon$ in a compact and precise trigonometric series expansion. Then we converted the true pole offsets observed by VLBI for 1979-2000 to the offests in the two precession angles, $\bar{\psi}$ and $\bar{\epsilon}$. As their first approximation, we used η_A and ε_A of Williams (1994). From the converted offsets, we determined the best-fit polynomial expressions of the corrections to the approximations as well as the equinox correction. We judged from a weighted least square method that linear corrections are sufficient. Thus we determined the polynomial expressions of $\bar{\psi}$ and $\bar{\epsilon}$ as well as the equinox correction E to the IAU 1976 formulation. Combining these with the above formulas of planetary precession, we determined the mean celestial pole offset at J2000.0 as

$$\bar{X}_0 = -(17.12 \pm 0.01)\,\text{mas}, \bar{Y}_0 = -(5.06 \pm 0.02)\,\text{mas}.$$

By shifting the base reference frame from the ICRF to the mean equator and equinox at J2000.0, we derived the best-fit polynomials of the classic precession quantities, $\sin\pi_A \sin\Pi_A$, $\sin\pi_A \cos\Pi_A$, π_A, Π_A, p_A, ψ_A, ω_A, χ_A, ϵ_A, ζ_A, z_A, and θ_A. They are provided in Table 1 of Fukushima (2003) as well as all the existing formulas given in the literature.

As a by-product, we estimated the speed of general precession in longitude at J2000.0 as

$$p = (5028.7955 \pm 0.0003)'' / \text{Julian century},$$

the mean obliquity at J2000.0 in the inertial sense as

$$(\epsilon_0)_I = (84381.40621 \pm 0.00001)'',$$

and that in the rotational sense as

$$(\epsilon_0)_R = (84381.40955 \pm 0.00001)''.$$

Also, by adopting a best estimate of the theoretical value of the geodesic precession,

$$p_g = (1.9196 \pm 0.0003)''/\text{Julian century},$$

we determined the dynamical flattening of the Earth from the precession constant as

$$H_d = (3.2737804 \pm 0.0000003) \times 10^{-3}.$$

From the classic polynomial expressions derived in the above, we evaluated the effect of the sense in defining the ecliptic. The resulting polynomial forms of the new precession angles in both the rotational and inertial senses are, as well as the derived classic precession quantities, summarized in [4]. These constitute a new set of the fundamental expressions of the precession quantities.

If the concern is the minimization of modification of the pre-2003 IAU formulation, the readers may use the updated classic quantities as well as the frame adjustment at the epoch.

Note that, although we have called $\bar{\psi}$ and $\bar{\epsilon}$ the luni-solar precession in this article, they are meant to include the effect of the geodesic precession. In spite of our presentation of precession quantities in both senses, one should understand that those in the inertial sense are of primary nature and the expressions in the rotational one were derived from them.

Refrences

[1] Williams, J. G.: 1994, *Astron. J.*, **108**, 711.
[2] Fukushima, T.: 2003, *Astron. J.*, **126**, 494.
[3] Harada, W., and Fukushima, T.: 2004, *Astron. J.*, **127**, 531.
[4] Shirai, T., and Fukushima, T.: 2001, *Astron. J.*, **121**, 3270.